The synthesis of equivalence digital filters for tandem decimation on base I/Q-demodulation

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The article describes the implementation of methodology for synthesis of equivalent digital filters for tandem decimation on base I/Q-demodulation.

Keywords: Tandem decimation, I/Q-demodulator, digital filter

INTRODUCTION

According to the concept of the digital I/Q-demodulation [1] can use of combinations of different schemes of signal processing [2, 3]. As example of this is the implementation of tandem decimation on base I/Q-demodulator in I\textsuperscript{st} stage of digital signal processing after of analog-to-digital-converter (Fig. 1), which was proposed in [4]. In this article the new methodology for synthesis of equivalent digital filters for tandem decimation on base I/Q-demodulation are consider.

MAIN

The key feature of the tandem scheme [4] is a two-stage signal processing:

1 stage - using of I/Q-demodulation as example with coefficients, which was proposed in [1]:

\[ U_i^c = U_i^t - 1 \cdot U_i^{t+2} + 15 \cdot U_i^{t+4} - 5 \cdot U_i^{t-6}. \]

(1)

where \( t \) – serial number reference ADC,

\( U_i^{c} \) - the quadrature components of voltages;

2 stage - using the results of the specified operation of the weight signal processing as the input array of quadrature samples for dual-channel operation of decimation samples ADC in accordance with the expressions:

\[ W_i^c = \sum_{\gamma = 0}^{\frac{M-1}{2}} \left \{ U_i^c \cdot \cos \left ( \omega_0 \gamma \right ) - U_i^s \cdot \sin \left ( \omega_0 \gamma \right ) \right \}, \]

\[ W_i^s = \sum_{\gamma = \frac{M}{2}}^{\frac{M+1}{2}} \left \{ U_i^c \cdot \cos \left ( \omega_0 \gamma \right ) - U_i^s \cdot \sin \left ( \omega_0 \gamma \right ) \right \}, \]

(2)

where \( \mathbf{U}^{(c,s)} \) – quadrature components of the signal at the output of the digital quadrature demodulation operation (“sliding window”), \( \gamma = 0, Y \cdot 1 \) – the sequence number of strobe, \( M \) – number of samples ADC, over which decimation operation is performed, \( \Omega_0 \) – the central frequency of the decimator filter, \( \tau \) – discretization period of ADC.

It is important to note that the discretization period of analog-to-digital converter (ADC) \( \tau \) should be selected so as to provide a ratio of the following form:

\[ f = \frac{4(2i+1)}{\tau}, \]

where \( f \) is the central frequency of the signal; \( i \) is the number of temporary count, \( i = 0, 1, 2, \ldots \)

The specified two-stages of processing is a drawback of the tandem scheme, making complicated its hardware implementation. Therefore, there is a benefit in replacing the signal processing in a tandem decimator equivalent weight of processing at fixed time intervals - gates. In the case of tandem decimator, which includes I/Q-demodulator of even order using P coefficients, the sample length of counts which are necessary for the formation of the response are equivalent digital filter for the duration of the strobe M of ADC counts, equal \( \sum = P \cdot M - 1 \), where \( P \) - dimension I/Q-demodulator. For example, for \( P=6, M=4 \) is \( P \cdot M - 1 = 6 \cdot 3 = 9 \) of counts.

Fig. 1. The tandem demodulator structure

General analytical record of the quadrature components responses of the equivalent options of the tandem decimation for different orders of the I/Q-demodulator and the duration of the strobe M might be written in the form of vector-matrix. Matrix uses a vector-row which elements are the weight coefficients of the demodulator that used the formation of the quadrature components in the first stage of the tandem scheme and the right oriented wedge-shaped matrix of units corresponding to the weight multipliers of the decimator in the second cascade of the tandem by not taking into account the sign. The number of units in the row of the matrix is equal to half the duration of the strobe and all
other elements are zero. The number of rows in the wedge-shaped matrix has been determined by \( P \), and the number of columns equal to the number of voltage counts are used to generate the quadrature responses (the I-component is formed by readings with even numbers, and Q-component – with odd numbers, however this is not a dogma). As an example, let’s consider the simplest case of calculating the equivalent digital filter coefficients for tandem scheme based on 5-counts I/Q-demodulator (\( P=5 \)) with coefficients \( \{4; 4\}, \{1; 6; 1\} \) [3] and decimation for the duration of the strobe \( M=2 \) of counts of ADC:

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix} = \{5; 10; 1\}
\]

Similarly, for the same 5-counts I/Q-demodulator (\( P=5 \)) with coefficients \( \{4; 4\}, \{1; 6; 1\} \) [3] and decimation for the duration of the strobe \( M=4 \) of counts of ADC will have:

\[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0
\end{pmatrix} = \{1; 11; 15; 5\}
\]

As can be seen, the resulting set of coefficients \( \{1; 11; 15; 5\} \) coincides with weight multipliers I/Q demodulator, shown in (1). Thus, it was shown that 8-counted I/Q-demodulator, which proposed in [1], can be obtained on the basis of sequentially connected 5-counted I/Q-demodulator and the decimator with the strobe duration of 4 samples of ADC.

On the other hand, this result allows us to use for decimation counts of ADC the new procedure of decimation which having a low level of side lobes of the amplitude-frequency characteristics. As an example, on Fig.2 shown amplitude-frequency characteristics (AFC) of 5-counts I/Q-demodulator with coefficients \( \{4; 4\}, \{1; 6; 1\} \) (thin solid line), a conventional decimator with the duration of the 8-counted strobe (the dotted line) and tandem of the 8-counted decimator on base 5-counted I/Q-demodulator in 1st stage.

Mentioned 5-counted I/Q-demodulator can will syntheses on base of 4-readings I/Q-demodulator whit coefficients \( \{1; 3\} \), which connected to decimator whit duration of strobes only 2 readings of ADC (for \( P=4, M=2 \) we have \( P+M-1=4+2-1=5 \) readings).

According to the above-described method, coefficients of 5-readings I/Q-demodulator should be formed in the following way:

\[
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix} = \{4; 4\}
\]

The set of weight multipliers \( \{1; 3\} \) of 4-readings I/Q-demodulator characterized in that their amount is not equal to zero, which is not allowed to get “zeros” of AFC. Therefore, AFC of the resulting 5-counted demodulator also has no zeros, since the equality to zero of the alternating sign sum of the coefficients must be performed in both quadrature components. However, as follows from the graphs in Fig. 2, the usage of the second cascade of a decimator that have zeros in the AFC, allows us also to obtain tandem demodulator with zeros in AFC.

In order to increase the suppression of side lobes AFC tandem demodulator, it is necessary to increase the order I/Q-demodulator in the 1st stage. The proposed method allows us to synthesize such demodulators. As an example, process of the 7-counted I/Q-demodulator synthesis can be considered, which is an equivalent tandem scheme, which is included in the first cascade of
4-counted demodulator and in the second cascade of
decimator with the 4 counts strobe duration of ADC (for
P=4, M=4 we have P+M-1=4+4-1=7 counts).
The equivalent demodulator coefficients are calculated
according to the expressions:
a) \[
\begin{pmatrix}
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0
\end{pmatrix}
\] = \[
\begin{pmatrix}
1 & 7 & 7 & 1
\end{pmatrix}
\]

б) \[
\begin{pmatrix}
1 & 1 & 0 \\
1 & 1 & 0 \\
0 & 1 & 1 \\
1 & 1 & 0
\end{pmatrix}
\] = \[
\begin{pmatrix}
4 & 8 & 4
\end{pmatrix}
\]

The resulting set of coefficients \{1;7;7;1\} and \{4;8;4\} allows to obtain a zero value for alternating sign sum of
the weighting multipliers, that generates zeros in AFC. Fig. 3 shows such an AFC 7-counted I/Q-demodulator (thin line) and a conventional decimator with the duration of the 8-counted strobe (the dashed line with high levels side lobes of AFC) and tandem 8-counted decimator on base 7-samples I/Q-demodulator in 1st cascade. There is noticeable the decrease in maximum of side lobes in comparison with Fig. 2.

Fig. 3. The AFC plots for I/Q demodulators

The resulting variant of 7-counted I/Q-demodulator notable because on its basis it is possible to synthesize the already mentioned 8-counted demodulator with quadrature responses (1). For that it is necessary to synthesize the equivalent digital filter corresponding to tandem demodulator by using of 7-counted demodulator in the first stage and 2-counted decimator (M=2) – in the second stage (for P=7 and M=2 we have P+M-1=7+2-1=8 readings). The synthesis of the corresponding coefficients can be made in the form of:

a) \[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\] = \[
\begin{pmatrix}
4 & 8 & 4 & 1 & 7 & 7 & 1
\end{pmatrix}
\] = \[
\begin{pmatrix}
1 & 1 & 1 & 5 & 1 & 5 & 1
\end{pmatrix}
\]

b) \[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0
\end{pmatrix}
\]

CONCLUSIONS
Thus, the proposed the methodology allows to synthesize as a new decimation filters of even and odd orders, and new I/Q-demodulators, which are the equivalent of tandem circuits digital filtering. The achieved suppression of the side petals allows the AFC to recommend an appropriate pre-processing for demodulation of signals with discrete non-orthogonal frequency division by multiplexing (N-OFDM) proposed in [5, 6]. This will allow to increase the spectral effectiveness of such signals for solving problems of communication when the number of subcarriers 32 and more.

REFERENCES