# CONTENTS

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# EXTENSION OF THE N-OFDM METHOD TO THE CASE OF ORTHOGONALLY POLARIZED SIGNALS

V. I. Slyusar and A. A. Zinchenko

*TsNII, Kiev, Ukraine*

**The paper considers a data transmission method in radio relay communication systems based on the use of nonorthogonal frequency discrete modulation (N-OFDM) of signals in combination with their orthogonal polarization. Synthesis of demodulation procedure of N-OFDM signals is conducted with regard for the presence of cross-polarization interference. In order to analyze the potentialities of the procedure synthesized, it is proposed to employ the well-known procedure of calculating the Cramer-Rao lower bound for variance of potentially achievable errors at measurement of quadrature components of signal amplitudes.**

One of promising directions of development in radio relay systems is the use of orthogonal frequency discrete modulation (OFDM) for improving their traffic capacity. The method is widely used in WiMAX-networks and ensures communications at direct visibility ranges (tens of kilometers). Further development of this approach is the method of nonorthogonal frequency discrete modulation (N-OFDM) [1, 2] permitting to multiplex the spectral band occupied by the signal, to tune away adaptively from narrow-band interference, and effectively operate under conditions of the Doppler shift of carrier frequencies. However, the works on development of the N-OFDM method up till now have been conducted with respect to a single-type polarization of the signals radiated. On the other hand, the use of two orthogonal polarizations of radiation makes it possible to almost double the capacity of a radio link. Such solutions are already known for the OFDM method [3].

The purpose of this paper is to elaborate the N-OFDM method by using a dual-polarization signal with nonorthogonal carriers.

Particularly, the task of our investigation is to synthesize demodulation procedures of N-OFDM signals with regard for the impact of cross-polarization interference.

It is assumed that the principle of generation of dual polarized signal packet with nonorthogonal carriers in the transmitter is reduced to usage of two independent quadrature channels of digital-to-analog conversion, each loaded with its own radiator. The subcarriers undergo the quadrature-amplitude modulation (QAM) in accordance with the message transmitted. Reception of the signals is performed by a similar antenna with a polarization selector and subsequent quadrature analog-to-digital conversion in each of the polarization channels.

Demodulation of the signals is performed under the assumption that exact frequencies of subcarriers are known. In order to take the cross-polarization interference into account, adaptive reduction of the QAM-modulation level is applied. In this case, estimation of the level of cross-polarization components is conducted by pilot-signals radiated both from the transmitter to receiver and in the opposite direction.

The synthesis of demodulation procedure will be performed using the method of least squares. Assume that in the general case the radiation on orthogonal polarizations using the N-OFDM method is conducted at non-coinciding subcarriers, while the number of subcarriers themselves in different polarizations is the same. Assume also that the levels of cross-polarization interference depend on frequencies of signals and are characterized by known relationships for each harmonic component. Under these assumptions the response of the receiving system after the synthesis of orthogonal frequency filters by the fast Fourier transform (FFT) can be presented in the matrix form:

$$U = F \cdot A + n \tag{1}$$

where $U$ is the block-vector of complex voltages at outputs of $R$ FFT-filters in two polarization channels of reception ($H$ is horizontal polarization, $V$ — the vertical one):

$$U = \begin{bmatrix} U_H \\ U_V \end{bmatrix}$$

$$U_H = [\, \dot{U}_{H1} \quad \dot{U}_{H2} \quad \ldots \quad \dot{U}_{HR} \,]^T, \quad U_V = [\, \dot{U}_{V1} \quad \dot{U}_{V2} \quad \ldots \quad \dot{U}_{VR} \,]^T,$$

$$F = \begin{bmatrix} F_H & F_{HV} \\ F_{VH} & F_V \end{bmatrix} = \begin{bmatrix} F_{H_1}(\omega_{1H}) & \cdots & F_{H_1}(\omega_{MH}) & F_{HV_1}(\omega_{1V}) & \cdots & F_{HV_1}(\omega_{MV}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{H_R}(\omega_{1H}) & \cdots & F_{H_R}(\omega_{MH}) & F_{HV_R}(\omega_{1V}) & \cdots & F_{HV_R}(\omega_{MV}) \\ F_{VH_1}(\omega_{1H}) & \cdots & F_{VH_1}(\omega_{MH}) & F_{V_1}(\omega_{1V}) & \cdots & F_{V_1}(\omega_{MV}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ F_{VH_R}(\omega_{1H}) & \cdots & F_{VH_R}(\omega_{MH}) & F_{V_R}(\omega_{1V}) & \cdots & F_{V_R}(\omega_{MV}) \end{bmatrix} \tag{2}$$

$F$ is the block matrix of amplitude-frequency responses (AFR) of $R$ FFT-filters of two polarization channels of receiver ($H$ is horizontal polarization, $V$ — the vertical one) at the main and cross-polarization components for $M$ frequencies of subcarriers. $F_{HV_r}(\omega_{mV})$, $F_{VH_r}(\omega_{mH})$ is the value of the AFR of the $r$th FFT-filter at the $m$th signal frequency for cross-polarization components (in the general case, $\omega_{mH} \neq \omega_{mV}$, $F_{H_r}(\omega_{mH})$, $F_{V_r}(\omega_{mV})$ is value of the AFR of the $r$th FFT-filter at the $m$th signal frequency for horizontal and vertical polarizations of reception, respectively,

$$A = [\, \dot{a}_{H1} \quad \dot{a}_{H2} \quad \ldots \quad \dot{a}_{HM} \quad \dot{a}_{V1} \quad \dot{a}_{V2} \quad \ldots \quad \dot{a}_{VM} \,]^T,$$

$\dot{a}_{Hm}$, $\dot{a}_{Vm}$ is the complex amplitude of the $m$th signal for the horizontal and vertical polarizations, respectively; and $\dot{n}$ is the block-vector of measurement noise.

Provided we cannot adopt the hypothesis of invariance of cross-polarization interference with respect to the signal frequency, we can write:

$$F_{HV_r}(\omega_{mV}) = q_{HV}(\omega_{mV}) \cdot F_{V_r}(\omega_{mV}) \quad \text{and} \quad F_{VH_r}(\omega_{mH}) = q_{VH}(\omega_{mH}) \cdot F_{H_r}(\omega_{mH}),$$

where $q_{HV}(\omega_{mV})$, $q_{VH}(\omega_{mH})$ are frequency-dependent coefficients of the polarization coupling in the event of its nonreciprocity, i.e., $q_{HV}(\omega_{mV}) \neq q_{VH}(\omega_{mH})$ even at $\omega_{mV} = \omega_{mH}$.

As a matter of fact, the coefficients $q_{HV}(\omega_{mV})$, $q_{VH}(\omega_{mH})$ characterize cross-polarization isolation (XPI) of the communication channel. Their values lie in the interval from 0 to 1. Transition from $q_{HV}(\omega_{mV})$, $q_{VH}(\omega_{mH})$ to the standardized notion XPI is conducted in accordance with the expressions [4]:

$$\text{XPD}_{HV} = 10\lg \frac{1}{q_{HV}(\omega_{mV})}, \quad \text{XPD}_{VH} = 10\lg \frac{1}{q_{VH}(\omega_{mH})}.$$

The standard level of cross-polarization isolation in terms of power is usually the XPI value from 20 to 30 dB. In high-quality systems this index may reach 40 dB or more. In our case the coefficients $q_{HV}(\omega_{mV})$, $q_{VH}(\omega_{mH})$ are calculated via voltage ratios of signals of the cross and main polarizations at the outputs of frequency FFT-filters. Because of this, the above XPI values in dB should be doubled.

Given the word size of analog-to-digital (ADC) or digital-to-analog (DAC) converters, the maximum level of the cross-polarization interference manifesting itself at the resonance frequency of FFT-filter can be easily estimated. Then it is assumed that due to QAM-modulation, the maximum value of the symbol level of N-OFDM signal corresponds to the upper bound of the word size of the ADC or DAC (without regard for the sign bit). For example, in the case of a 16-bit ADC the boundary value of voltage of the signal received is 90 dB (15 bits) with respect to the quantum of ADC. Hence, at

XPI = 40 dB in terms of power, which corresponds to the magnitude of coefficients $q_{HV}(\omega_{mV})$, $q_{VH}(\omega_{mH})$ minus 80 dB, the elevation of the voltage of cross-polarization interference over the ADC quantum may reach 10 dB. An advantage of the N-OFDM modulation, as compared with conventional OFDM, consists in the opportunity of additional reduction of the parasitic cross-components due to nonorthogonal allocation of frequencies of the signals. In this case, one of the polarizations may involve signals that have OFDM modulation, while the other — signals with N-OFDM modulation with allocation of subcarriers in intersection points of AFR of FFT-filters. An appropriate option is presented in Fig. 1, where solid arrows denote allocation of the signal frequencies, e.g., of vertical polarization (OFDM), while the dashed ones (top row) — horizontal polarization (N-OFDM). The AFR of FFT-filters are shown under the arrows. In a more general case, the scheme of N-OFDM modulation is used in both polarization components and, moreover, the rated values of subcarriers are set for each polarization in such way that their frequencies do not coincide with the frequencies of cross-components.

Neglecting the frequency dependence of cross-polarization interference and its sensitivity on the direction of polarization coupling, we may presume that

$$F_{HV_r}(\omega_m)=q\cdot F_{V_r}(\omega_m) \quad \text{and} \quad F_{VH_r}(\omega_m)=q\cdot F_{H_r}(\omega_m),$$

i.e., the AFR matrix assumes the form:

$$F=\begin{bmatrix} F_H & q\cdot F_V \\ q\cdot F_H & F_V \end{bmatrix}=[F_H \quad F_V]\blacksquare\begin{bmatrix} 1 & q \\ q & 1 \end{bmatrix}, \tag{3}$$

where ▪ is symbol of the Chatri-Rao matrix product [5].

In the case of negligible cross-polarization interference, the coefficient $q$ can be set equal to zero. Hence, we obtain:

$$F=[F_H \quad F_V]\blacksquare\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} F_H & 0 \\ 0 & F_V \end{bmatrix}. \tag{4}$$

Such assumption is valid, for example, at ADC (DAC) word size of 14 bit or less, provided XPI = 40 dB, when the level of cross-polarization interference does not exceed the quantum of ADC. In addition, this model may be accepted, if the magnitude of the voltage of cross-components is less than the admissible level of decoding errors of messages, determined by required probability of error-free data transmission.

Using the matrix relationships (1), one can easily estimate the vector of complex amplitudes of signals referred to ADC outputs of the receiving channel, which is necessary for demodulation of messages. To this end, using the method of least squares, we must minimize the functional of discrepancies:

$$L=\{\dot{U}-F\dot{A}\}^{*}\{\dot{U}-F\dot{A}\}=\min. \tag{5}$$

Obviously, the appropriate estimate of the vector of amplitudes $A$ can be found in the form:

$$\hat{A}=\{F^{T}F\}^{-1}F^{T}\dot{U}. \tag{6}$$

Hence, the sought quadrature components of amplitudes of signals at the ADC output can be calculated by the following relationships:

$$A^{c}=\mathrm{Re}(\{F^{*}F\}^{-1}\cdot F^{*}\cdot\dot{U}), \quad A^{s}=\mathrm{Im}(\{F^{*}F\}^{-1}\cdot F^{*}\cdot\dot{U}), \tag{7}$$

where

$$A^{c}=[\,a_{H1}^{c} \quad a_{H2}^{c} \quad \ldots \quad a_{HM}^{c} \quad a_{V1}^{c} \quad a_{V2}^{c} \quad \ldots \quad a_{VM}^{c}\,]^{T},$$

$$A^{s}=[\,a_{H1}^{s} \quad a_{H2}^{s} \quad \ldots \quad a_{HM}^{s} \quad a_{V1}^{s} \quad a_{V2}^{s} \quad \ldots \quad a_{VM}^{c}\,]^{T}.$$

It is essential that matrices $F$ and vectors $A$, $U$ in any of interpretations (2)–(4) may be substituted into (7). Note that in this case we do not need to account the word size (number of bits) of FFT when calculating the maximum level of cross-polarization interference, since the latter is accumulated coherently, similar to legitimate signal, during the synthesis of FFT-filters. Moreover, procedure (6) makes it possible to estimate the original amplitude of signals at the input of FFT-operation rather than after it.

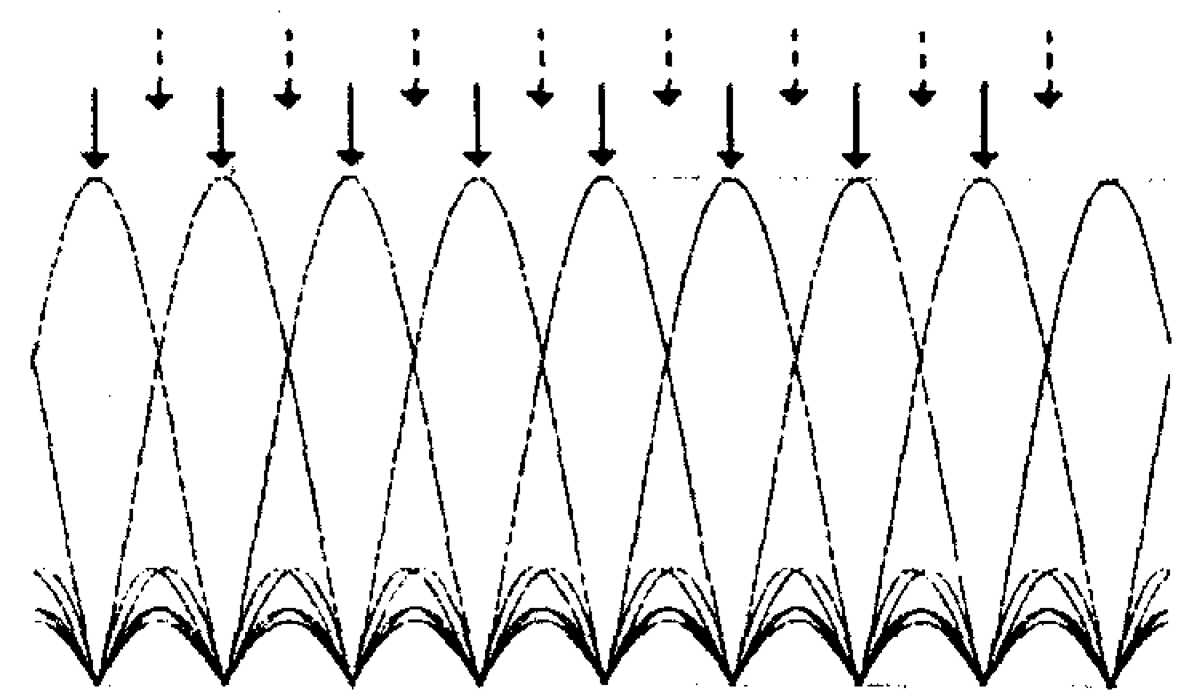

Fig. 1

Analysis of potentialities of the approach considered, permitting to choose the proper algorithm of QAM-modulation, can be performed by the known method of calculation of the Cramer-Rao lower bound (CRLB) for the potentially achievable error variance of the measurement of amplitude components [6]. The Fisher matrix required in this case can be calculated under the condition of uncorrelated Gaussian noise of orthogonal polarizations in quadrature components of voltages by the following formula:

$$I = \frac{1}{\sigma_{ns}^2} \cdot F^T F \qquad (8)$$

where $F$ is the AFR matrix of FFT-filters in two orthogonal polarizations presented in expressions (1)–(7), $\sigma_{ns}^2$ is the noise variance in quadrature components of voltages of signals (assumed to be equal in the quadrature channels of both polarizations).

The calculation results of CRLB by inversion of (8) for the case of the zero coefficient of cross-polarization $q$ and $q \neq 0$ confirm the obvious conclusion that in the absence of cross-polarization interference the potential accuracy of demodulation of messages will be higher than at $q \neq 0$ — even in the case of matched processing, which takes the known parameters of the cross-polarization coupling into account.

## REFERENCES

1. V. I. Slyusar and V. G. Smolyar, Izvestiya VUZ. Radioelektronika, Vol. 46, No. 7, pp. 30–39, 2003.
2. V. I. Slyusar and V. G. Smolyar, Izvestiya VUZ. Radioelektronika, Vol. 47, No. 4, pp. 53–59, 2004.
3. Vinko Erceg, Pitchaiah Soma, Daniel S. Baum, and Severine Catreux, IEEE Transactions on Wireless Communications, Vol. 3, No. 6, 2004, http://www.nari.ee.ethz.ch/commth/pubs/files/twc04.pdf.
4. R. U. Nabar, "MIMO Channel Models," Lecture, Communication Technology Laboratory, Zurich, http://www.nari.ee.ethz.ch/commth/teaching/mimo/lect3.pdf.
5. V. I. Slyusar, Izvestiya VUZ. Radioelektronika, Vol. 46, No. 10, pp. 15–26, 2003.
6. V. I. Slyusar, Izvestiya VUZ. Kibernetika i Sistemniy Analiz, No. 4, pp. 141–149, 1999.

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