X ANNIVERSARY INTERNATIONAL CONFERENCE ON ANTENNA THEORY AND TECHNIQUES ICATT’2015

Dedicated to 95 year jubilee of Prof. Yakov S. Shifrin

PROCEEDINGS

April 21 – 24, 2015
Kharkiv, Ukraine

Proceedings of 2015 X Anniversary International Conference on Antenna Theory and Techniques (ICATT)
IEEE Catalog Number CFP15540-PRT
ISBN 978-1-4799-8556-2

I/Q-DEMODULATION OF THE ODD ORDER

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Abstract

In this paper the advanced technique of I/Q demodulation of the odd order synthesis for any dimension is presented. Calculation of coefficients for a signal filtration is based on the common solution of the redefined and undefined system of the homogeneous sedate equations concerning independent variables.

Index Terms: I/Q-demodulator; analog-to-digital convertor (ADC); amplitude-frequency characteristic (AFC).

1. INTRODUCTION

Wide spreading of the digital signal processing in radars and communication means lays special stress on the problem of digital formation of the quadrature voltage components in the receiving segment of the relevant radio systems. Especially it is very important to solve the above mentioned problem in the case of one-channel analog-to-digit signal conversion. A digital method of the quadrature demodulation based on the usage of odd number of signal samples is presented below.

2. THEORY

In patent [1], a method of the even order I/Q demodulation is proposed, which differs by the linearity of the phase-frequency characteristics at a wide band of signal reception and by the integer-valued weighting coefficients that are calculated by solving a system of exponential equations. There is described the 4-reading I/Q demodulator with weighting coefficients {1; 3}, whose quadrature components (so-called "in phase" (I) and "quadrature" (Q)) are calculated in the following way:

$$I = u_1 - 3u_3\,;\; Q = -\left(3u_2 - u_4\right), \qquad (1)$$

where u_n is the voltage of signal reading at the output of the analog-to-data convertor at the nth moment of time.

Besides, described in [1] is an alternative 6-reading variant of the I/Q demodulator with coefficients {1; 10; 5}, which have following type of quadrature responses:

$$I = u_1 - 10u_3 + 5u_5\,; \quad Q = 5u_2 - 10u_4 + u_6\,. \quad (2)$$

Disadvantage of the I/Q demodulators (1), (2) presented in [1] is the absence of nulls in their amplitude-frequency characteristic that deteriorates the frequency selectivity of the quadrature formation procedure.

With regard to the 8-reading variant of the signal I/Q demodulation, considered in [2], the equations system for the weighting factors calculation should be formed taking into consideration equality of linear, quadratic, cubic, etc. components of signals. For the I and Q values all these components are equal:

$$\begin{aligned} a + 3b + 5c + 7d &= 2h + 4g + 6f + 8e; \\ a + 3^2 b + 5^2 c + 7^2 d &= 2^2 h + 4^2 g + 6^2 f + 8^2 e; \\ a + 3^3 b + 5^3 c + 7^3 d &= 2^3 h + 4^3 g + 6^3 f + 8^3 e; \\ a + 3^4 b + 5^4 c + 7^4 d &= 2^4 h + 4^4 g + 6^4 f + 8^4 e. \end{aligned} \qquad (3)$$

Numbers near *a, b, …, e* coefficients in system (3) indicate the order number of the signal voltage readings whose numeration starts from one. Therein the sum of odd readings is placed on the left side of equation and the sum of even readings is on the right side.

From all the (3) solution sets, of interest for us are integer-valued weighting factors satisfying condition for their alternating sum to be equal to zero. The corresponding 8-reading procedure of the signals I/Q demodulation, which uses such weighting factors, looks like [2]:

$$\begin{aligned} I &= u_1 - 11u_3 + 15u_5 - 5u_7\,; \\ Q &= 5u_2 - 15u_4 + 11u_6 - u_8\,. \end{aligned} \qquad (4)$$

For the synthesis of the I/Q demodulators of the even order of high dimensionality it is proposed to use the updated procedure of synthesis method, the initial point of which is the formation of the system of homogeneous equations for calculation of the filtration coefficient of the type [3]:

$$\begin{aligned} &\left(2^p N^p - 1\right)a_1 + \ldots \cdot \left(2^p (N-n+1)^p - (2\mathrm{n}-1)^p\right)a_n + \ldots \\ &\ldots + \left(2^p - (2\mathrm{N}-1)^p\right) \cdot a_N = 0 \end{aligned} \qquad (5)$$

where $p = \overline{1,N}$ is number of equation, N is the order of the equation system.

This paper presents a new class of the I/Q demodulators that have an odd order, i.e. which use for the response formation an odd number of the signal time readings. In this case, a necessary set of weighting coefficients can be obtained by solving the exponential equations system that differs from (3), (5) by us-

978-1-4799-8557-9/15/$31.00 2015 IEEE

ing odd and even number of summands in the left and right parts of the respective equations.

For example, the simplest I/Q demodulator of the odd order is 3-reading variant whose coefficients are calculated by the following equation:

$$2a = 1b + 3b \,. \qquad (6)$$

Solving (6) as it is, we obtain: a=2C; b=C. For example, if C=1, quadrature components should be formed in the following way:

$$I = 2u_2 \,;\; Q = u_1 - u_3 \,. \qquad (7)$$

The analog of equations system (3), which allows obtaining coefficients of the 5-reading demodulator, is the following set of equations:

$$2a + 4a = 1b + 3c + 5b \,;$$
$$2^2 a + 4^2 a = 1b + 3^2 c + 5^2 b \,. \qquad (8)$$

This I/Q pair gives a correct interpolation of a linearly or quadratically varying signal. Solution of (8) in the integer range is a=4C; b=C; c=6C. Thus, if C=1,

$$I = 4u_2 - 4u_4 \,;\; Q = u_1 - 6u_3 + u_5 \,. \qquad (9)$$

If to use the underdetermined system, leaving in (8) only one equation, we obtain:

$$2a + 4a = 1b + 3c + 5b \,. \qquad (10)$$

A region of the (10) solutions covers many coefficient sets being determined by two independent variables C[1] and C[2]:

a=C[1]; b=C[2]; c=2 C[1]-2 C[2]. (11)

In order for negative coefficient values not to be obtained, the condition 2C[1]>2C[2] should be satisfied. Taking this into consideration and if C[1]=2 and C[2]=1, we obtain a=2, b=1, c=2. Such a set of quadrature multipliers allows forming responses of the I/Q demodulator of the following type:

$$I = 2u_2 - 2u_4 \,;\; Q = u_1 - 2u_3 + u_5 \,. \qquad (12)$$

It is not difficult to obtain a=3, b=1, c=4 if C[1]=3 and C[2]=1, and respectively:

$$I = 3u_2 - 3u_4 \,;\; Q = u_1 - 4u_3 + u_5 \,. \qquad (13)$$

Similarly, if C[1]=4 и C[2]=1, the set of quadrature multipliers is a=4, b=1, c=6, to which algorithm (9) of formation of quadrature corresponds.

In this case, the I-component is formed by readings with even numbers, and Q-component – with odd numbers, however this is not a dogma. The above mentioned list of I/Q demodulation responses variants can be extended further, but of interest for practical purposes in a set of weighting coefficients having a minimum spread in values and providing zeroes availability in amplitude-frequency characteristic (AFC).

To obtain the 5-reading demodulator coefficients with zero values of AFC, it is necessary to add to (10) an equation that describes condition of the alternating coefficient sum nullification. While forming such an equation, the coefficients of the quadrature components with an odd number of summands should be used, and in this case Q-quadratures are the following:

$$2a + 4a = 1b + 3c + 5b \,;$$
$$b - c + b = 0 \,. \qquad (14)$$

A unique integer-valued solution that satisfies this condition is: a=2C; b=C; c=2C, that corresponds to the above mentioned variant of quadrature formation component (12) if C=1. The AFC plots that correspond to the 3-reading and 5-reading demodulator variants are seen in Fig.1

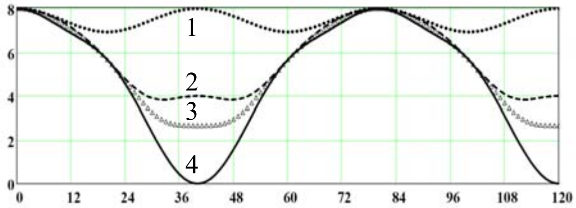

Fig.1. AFC plots for I/Q demodulators (7), (9), (12), (13), where number “1” is for AFC procedure (7), “2” for AFC demodulator (9), “3” for AFC (13) and “4” for AFC (12).

The proposed method can yield I/Q-filters of different order, and hence of different performance.

The calculation of the coefficients of 7-sampling quadrature demodulation version is based on the solution of the equation system of the following type:

$$2a + 4b + 6a = 1c + 3d + 5b + 7c \,;$$
$$2^2 a + 4^2 b + 6^2 a = c + 3^2 d + 5^2 b + 7^2 c \,;$$
$$a - b + a = 0 \,. \qquad (15)$$

The sought solution of (15) is in the multipliers

a=4 C[1]; b=8 C[1]; c=C[1]; d=7 C[1].

At C[1]=1, it is easy to obtain a=4; b=8; c=1; d=7, what allows writing the relation for the I/Q-component formation as

$$I = 4u_2 - 8u_4 + 4u_6 \,;\; Q = u_1 - 7u_3 + 7u_5 - u_7 \,. \qquad (16)$$

Transition to the underdetermined system of equations:

$$2a + 4b + 6a = 1c + 3d + 5b + 7c \,;$$
$$a - b + a = 0 \,, \qquad (17)$$

provides the possibility to obtain an extensive set of variants of the weighting factors which are expressed in basis of variables C[1] and C[2]:

a=C[1]; b=2C[1]; c=C[2]; d=2C[1]-C[2]. (18)

At C[1]=1, C[2]=1, it follows from (18) that a=1; b=2; c=1; d=1, and:

$$I = u_2 - 2u_4 + u_6 \,;\; Q = u_1 - u_3 + u_5 - u_7 \,. \qquad (19)$$

Values of some other sets of weighting coefficients are seen in Table.1

Plots of AFC of the synthesized 7-reading demodulators and AFC of the 5-reading version are presented in Fig.2.

As should be expected, the transition to the seventh order demodulator allows improving the frequency selectivity as compared with the 5-reading one. The final choice of a specific group of multipliers is the compromise problem that can be solved on the basis of the AFC shape analysis taking into consideration the necessity to keep its smooth inclining and to obtain max dip in the domain of zero values. The mathematical formalization of this task merits a closer examination.

Table 1. Coefficients of 7-sampling I/Q-demodulation

C[1]	C[2]	a	b	c	d
2		2	4		3
3		3	6		5
4		4	8		7
5	1	5	10	1	9
6		6	12		11
7		7	14		13
8		8	16		15

Even more degrees of freedom while searching the mentioned compromise provide transition to 9-reading I/Q-demodulators. In this case, 4 signal readings with even order numbers and 5 readings of odd order are used for quadrature component formation. The corresponding set of equations is the following:

$$2a+4b+6b+8a=c+3d+5e+7d+9c;$$
$$2^2a+4^2b+6^2b+8^2a=c+3^2d+5^2e+7^2d+9^2c;$$
$$2^3a+4^3b+6^3b+8^3a=c+3^3d+5^3e+7^3d+9^3c;$$
$$2^4a+4^4b+6^4b+8^4a=c+3^4d+5^4e+7^4d+9^4c;$$
$$c-d+e-d+c=0. \qquad (20)$$

On the set of integer numbers its solution can be represented as a=6C; b=26C; c=C; d=16C; e=30C, whence, at C=1, it follows that:

$$I=6u_2-26u_4+26u_6-6u_8;$$
$$Q=u_1-16u_3+30u_5-16u_7+u_9. \qquad (21)$$

To obtain a set of coefficients with less data scattering is possible when excluding from (20) the equation of the 4th degree with transition to the underdetermined variant of system:

$$2a+4b+6b+8a=c+3d+5e+7d+9c;$$
$$2^2a+4^2b+6^2b+8^2a=c+3^2d+5^2e+7^2d+9^2c;$$
$$2^3a+4^3b+6^3b+8^3a=c+3^3d+5^3e+7^3d+9^3c;$$
$$c-d+e-d+c=0. \qquad (22)$$

Solution set (20) is calculated from the following relations:

a=C[1]; b=7C[1]+16C[2]; c= -C[2];
d=4C[1]+8C[2]; e=8C[1]+18C[2]. (23)

The analogue of system (20), having sololution in the basis of three independent variables C[1], C[2], C[3], looks like:

$$2a+4b+6b+8a=c+3d+5e+7d+9c;$$
$$c-d+e-d+c=0. \qquad (24)$$

The sought coefficients are expressed from (24) through three variables as:

a=C[1]; b=C[1]+2C[2]; c=C[3]; d=C[1]+C[2];
e=2(C[1]+C[2]-C[3]).

The obtained results show that the increase in the number of independent variables, being used for coefficient demodulation calculation, allows decreasing a

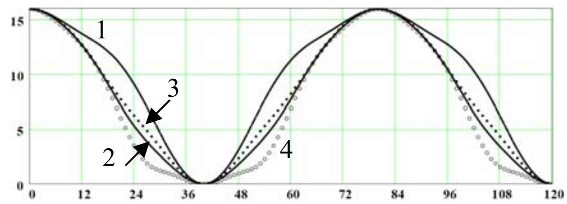

Fig. 2. AFC plots for the 7-reading I/Q-demodulator in comparison with those for 5-reading procedure (12), where number "1" for AFC procedure (12), "2" for AFC of 7-sampling demodulator with a set of coefficients {a=3; b=6; c=1; d=5}; "3" for AFC procedure (16); "4" for AFC of the coefficients {a=2; b=4; c=1; d=3}.

spread (dynamic range) of values of weighting factors. Nevertheless, it should be noted that the occurrence of AFC side lobes is typical for the set of coefficients with a minimal dynamic range of values, but this is not always acceptable in practice.

Based on examined approaches to coefficient calculation of I/Q-filtration, the author obtained its family for higher order demodulation. In the first variant of 11-reading demodulator we obtain a=6; b=32; c=52; d=1; e=17; f=46, and, respectively:

$$I=6u_2-32u_4+52u_6-32u_8+6u_{10};$$
$$Q=u_1-17u_3+46u_5-46u_7+17u_9-u_{11}. \qquad (25)$$

3. Conclusion

The proposed method of analytical calculation of weighting coefficients of I/Q-demodulator of the odd dimension simplifies the problem of its synthesis. The further investigations should be focused on the analysis of phase errors of the examined demodulators, as well as on the comparative analysis of their properties, in order to make recommendations for practical use.

References

1. *Swedish patent* No. 501604 (C2). IPC G11C27/02, H03D1/22. Method and apparatus for sampling of electrical signals.// Arvidsson Erik Ragnar. – Appl. number SE19930002627. - Priority date: 1993-08-13. – Publ. date: 1995-03-27.
2. Eklund J.-E. and Arvidsson R. 1996, A Multiple Sampling, Single A/D Conversion Technique for Demodulation in CMOS.// *IEEE Journal of Solid-State Circuits*, **31**, 12, 1987 - 1994.
3. Slyusar V.I., Maljarchuk M.V, Bondarenko M.V. 2010, 'Method of synthesis of any dimension I/Q-demodulators', III *International scientific-techncal symposium "New the technologist in telicomunication"* (DUIKT-KARPATU'2010, with. Vushkiv). – Kiev: National Universety Information-Telecomunication Technologe. - 2-5 february - 53-55.- http://slyusar.kiev.ua/VYSHKIV_2010_2.pdf.