I/Q-Demodulation of the Odd Order

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***Abstract*—In this paper the advanced technique of I/Q demodulation of the odd order synthesis for any dimension is presented. A calculation of coefficients for a filtration of signals is based on the common decision of the redefined and undefined system of the homogeneous sedate equations concerning independent variables.** (Abstract)

Index Terms – **I/Q-demodulator; analog-to-digital convertor (ADC); amplitude-frequency characteristic (AFC)**

I. Introduction

Wide spreading of the digital signal processing in radars and communication means lays special stress on the problem of digital formation of the quadrature voltage components in the corresponding segment of the receiving radio systems. Especially it is very important to solve above mentioned problem in the field of single-channel analog-to-digital signal conversion. Digital method of the quadrature demodulation based on the usage of odd number of signal samples is presented below.

II. Theory

In patent [1] it was proposed a method of I/Q demodulation of the even order which differs by the linearity of the phase-frequency characteristics in a wide signal reception band and by the integer-valued weighting coefficients that are calculated by solving exponential equations system. At the same time it was described 4-sampling I/Q demodulator with weighting coefficients {1; 3}, and its quadrature components (a so-called “in phase” (I) and a so-called “quadrature” (Q)) were calculated in the following way:

$$I = u_1 - 3u_3\,;\ Q = -(3u_2 - u_4), \tag{1}$$

where u_n is a voltage of signal sample at the output of the analog-to-data convertor in the nth moment of time.

Moreover, in [1], it is described an alternative 6-sampling variant of the I/Q demodulator with coefficients {1; 10; 5} which have following type of quadrature responses:

$$I = u_1 - 10u_3 + 5u_5\,;\quad Q = 5u_2 - 10u_4 + u_6\,. \tag{2}$$

However, the disadvantage of the presented I/Q demodulators (1), (2) in [1] is the absence of zeroes in their amplitude-frequency characteristic that deteriorates the frequency selectivity of the quadrature formation procedure.

With regard to 8-sampling variant of the signal I/Q demodulation which was examined in [2], the equations system for the weighting factors calculation should be formed taking into consideration equality of a linear, quadratic, cubic, etc. components of signals. All these components are equal for the I and Q values:

$$\begin{aligned} a+3b+5c+7d &= 2h+4g+6f+8e; \\ a+3^2b+5^2c+7^2d &= 2^2h+4^2g+6^2f+8^2e; \\ a+3^3b+5^3c+7^3d &= 2^3h+4^3g+6^3f+8^3e; \\ a+3^4b+5^4c+7^4d &= 2^4h+4^4g+6^4f+8^4e. \end{aligned} \tag{3}$$

Numbers near *a, b, ..., e* coefficients in the system (3) indicate sequence number of the signal voltage samples and their numeration starts with one. At the same time, the sum of odd samples is placed on the left side of equation and the sum of even samples is on the right side.

From all solution sets (3), we are interested in integer-valued weighting factors that satisfy equal-zero condition of their alternating sum. Corresponding 8-sampling procedure of the signals I/Q demodulation, which uses such weighting factors, is the following [2]:

$$I = u_1 - 11u_3 + 15u_5 - 5u_7\,;\ Q = 5u_2 - 15u_4 + 11u_6 - u_8\,. \tag{4}$$

For the synthesis of the I/Q demodulators of the even order of high dimensionality it is proposed to use synthesis advanced technic, the starting point of which is the formation of the system of homogeneous equations for the filtration coefficient calculation of the type [3]:

$$\begin{aligned} &\left(2^p N^p - 1\right)a_1 + \ldots \left(2^p (N-n+1)^p - (2\mathrm{n}-1)^p\right)a_n + \ldots \\ &\ldots + \left(2^p - (2\mathrm{N}-1)^p\right)\cdot a_N = 0\,, \end{aligned} \tag{5}$$

where $p = \overline{1,N}$ is a number of equation, N is an order of the equation system.

This paper presents new class of the I/Q demodulators that have an odd order, i.e. which use for the response formation an odd number of the signal time samples. In this case essential set of the weighting coefficients may be obtained by solving exponential equations system that differs from (3), (5) by using odd and even number of summands in the left and right parts of the respective equations.

For example, the simplest I/Q demodulator of the odd order is 3-sampling variant and its coefficients are calculated by the following equation:

$$2a = 1b + 3b\,. \tag{6}$$

Solving (6) as it is, we obtain: a=2C; b=C. For example, if C=1, quadrature components should be formed in the following way:

$$I = 2u_2\,;\; Q = u_1 - u_3\,. \quad (7)$$

The analog of the equations system (3), which allows to obtain coefficients of the 5-sampling demodulator, is the following set of equations:

$$2a + 4a = 1b + 3c + 5b\,;$$
$$2^2 a + 4^2 a = 1b + 3^2 c + 5^2 b\,. \quad (8)$$

This I/Q pair gives a correct interpolation of a linearly or quadratically varying signal. Solution (8) in the integer range is a=4C; b=C; c=6C. Thus if C=1

$$I = 4u_2 - 4u_4\,;\; Q = u_1 - 6u_3 + u_5\,. \quad (9)$$

If to use underdetermined system, leaving in (8) only one equation, we obtain:

$$2a + 4a = 1b + 3c + 5b\,. \quad (10)$$

Solution region (10) covers many coefficient sets, which are determined by two independent variables C[1] and C[2]:

a=C[1]; b=C[2]; c=2 C[1]-2 C[2]. (11)

For us not to get negative coefficient values, the condition 2C[1]>2C[2] should be satisfied. Taking it into consideration and if C[1]=2 and C[2]=1, we obtain a=2, b=1, c=2. Such set of quadrature multipliers allow to generate responses of the I/Q demodulator of the following type:

$$I = 2u_2 - 2u_4\,;\; Q = u_1 - 2u_3 + u_5\,. \quad (12)$$

It is not difficult to obtain a=3, b=1, c=4 if C[1]=3 and C[2]=1, and respectively:

$$I = 3u_2 - 3u_4\,;\; Q = u_1 - 4u_3 + u_5\,. \quad (13)$$

Similarly, if C[1]=4 и C[2]=1, the set of quadrature multipliers is a=4, b=1, c=6, which corresponds to the quadrature formation algorithm (9).

In this case I-component is formed by odd numbers samples, and Q-component is formed by even numbers samples, however it is not a dogma. The above mentioned list of I/Q demodulation responses variants can be extended further, but for practical purposes, we are interested in a set of weighting coefficients that have minimum range of values spread and that ensure zeroes availability in amplitude-frequency characteristic (AFC).

To obtain 5-samplings demodulator coefficients with zeroes values of AFC, it is necessary to add to (10) an equation that describes condition of the nullification of the coefficient alternating sum. While forming such equation, the coefficients of the quadrature components with an odd number of summands should be used, and in this case Q-quadratures are following:

$$2a + 4a = 1b + 3c + 5b\,;$$
$$b - c + b = 0\,. \quad (14)$$

Unique integer solution that satisfies this condition is the following: a=2C; b=C; c=2C, that corresponding to above mentioned variant of the quadrature formation component (12) if C=1. AFC diagrams that correspond to 3-sampling and 5-sampling demodulator variants are seen in Fig.1. where number “1” is for AFC diagram procedure (7), “2” is for AFC demodulator (9), “3” is for AFC (13) and “4” is for AFC (12).

The proposed method can give I/Q-filters with another length, and thereby different performance.

Calculation of the coefficients of 7-sampling quadrature demodulation version is based on the solution of the equation system of the following type:

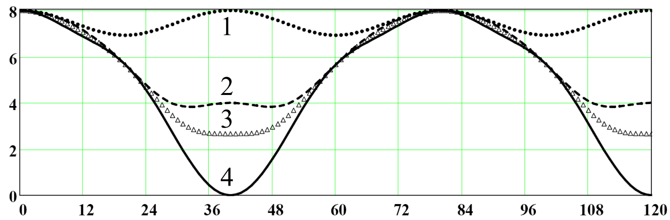

Fig. 1. Diagrams of the AFC I/Q demodulators (7), (9), (12), (13)

$$2a + 4b + 6a = 1c + 3d + 5b + 7c\,;$$
$$2^2 a + 4^2 b + 6^2 a = c + 3^2 d + 5^2 b + 7^2 c\,;$$
$$a - b + a = 0\,. \quad (15)$$

Following multipliers are the desired solution of (15):

a=4 C[1]; b=8 C[1]; c=C[1]; d=7 C[1].

If one takes into account that C[1]=1, it is easy to obtain a=4; b=8; c=1; d=7 and it allows to write relation for the I/Q-component formation in the following way:

$$I = 4u_2 - 8u_4 + 4u_6\,;\; Q = u_1 - 7u_3 + 7u_5 - u_7\,. \quad (16)$$

Transition to the undetermined system equations:

$$2a + 4b + 6a = 1c + 3d + 5b + 7c\,;$$
$$a - b + a = 0\,, \quad (17)$$

provides possibility to obtain an extensive set of variants of the weighting factors which are expressed in basis of variables C[1] and C[2]:

a=C[1]; b=2C[1]; c=C[2]; d=2C[1]-C[2]. (18)

As C[1]=1, C[2]=1 it follows from (18) that a=1; b=2; c=1; d=1, and:

$$I = u_2 - 2u_4 + u_6\,;\; Q = u_1 - u_3 + u_5 - u_7\,. \quad (19)$$

Values of some other sets of weighting coefficients are seen in Table.1

TABLE I. COEFFICIENTS OF 7-SAMPLING **I/Q-D**EMODULATION

C[1]	C[2]	a	b	c	d
2	1	2	4	1	3
3		3	6		5
4		4	8		7
5		5	10		9
6		6	12		11
7		7	14		13
8		8	16		15

Diagrams AFC of synthesized 7-sampling demodulators versus and AFC 5-sampling version are presented in Fig.2, where number “1” is for AFC procedure (12), “2” is for AFC of 7-sampling demodulator with a set of coefficients {a=3; b=6; c=1; d=5}; “3” is for AFC procedure (16); “4” is for AFC of the coefficients {a=2; b=4; c=1; d=3}.

As could be expected, the transition to demodulator of the seventh order allows to improve of frequency selectivity, comparing with 5-sampling one. The final choice of specific group of multipliers is a compromise problem, which can be solved on the base of AFC form analysis taking into consideration its gradual incline keeping and max. null

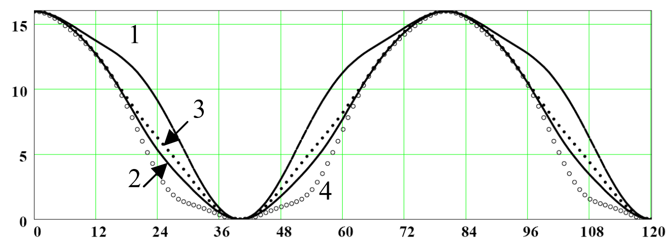

Fig. 2. Diagrams of AFC 7-sampling I/Q-demodulator in comparison with AFC of 5-sampling procedure (12).

obtaining in zero value area. The mathematical formalization of this task merits a closer examination.

Even more degrees of freedom while searching mentioned compromise provide transition to 9-sampling I/Q-demodulators. In this case 4 signal samples of even order and 5 sampling of odd order are used for quadrature component formation. The corresponding set of equations is the following:

$$2a+4b+6b+8a=c+3d+5e+7d+9c\,;$$
$$2^2a+4^2b+6^2b+8^2a=c+3^2d+5^2e+7^2d+9^2c\,;$$
$$2^3a+4^3b+6^3b+8^3a=c+3^3d+5^3e+7^3d+9^3c\,;$$
$$2^4a+4^4b+6^4b+8^4a=c+3^4d+5^4e+7^4d+9^4c\,;$$
$$c-d+e-d+c=0\,. \qquad (20)$$

On the set of integer numbers its solution can be represented as a=6C; b=26C; c=C; d=16C; e=30C, and if C=1 then:

$$I=6u_2-26u_4+26u_6-6u_8\,;Q=u_1-16u_3+30u_5-16u_7+u_9\,. \quad (21)$$

To obtain a set of coefficients with less data scattering is possible due to expulsion of equation of the 4th degree with transition to underdetermined variant of system from (20):

$$2a+4b+6b+8a=c+3d+5e+7d+9c\,;$$
$$2^2a+4^2b+6^2b+8^2a=c+3^2d+5^2e+7^2d+9^2c\,;$$
$$2^3a+4^3b+6^3b+8^3a=c+3^3d+5^3e+7^3d+9^3c\,;$$
$$c-d+e-d+c=0\,. \qquad (22)$$

The solution set (20) is calculated in the following way:

a=C[1]; b=7C[1]+16C[2]; c= -C[2];
d=4C[1]+8C[2]; e=8C[1]+18C[2]. (23)

Some of possible coefficient groups (23) are represented in Table II. In this case the solution C[1]=2, C[2]=-1 is inappropriate as it causes negative coefficient values a=2; b=-2; c=1; d=0; e=-2.

The analogue system (20), which is solved in the basis of three independent variables C[1], C[2], C[3], has the following type:

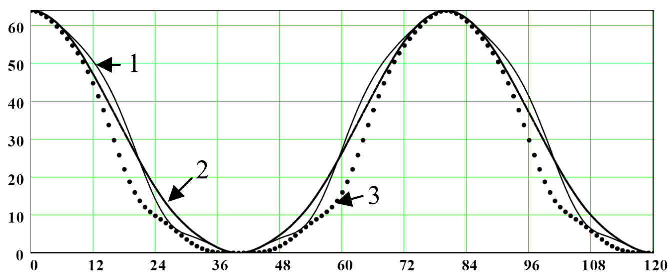

Fig. 3. Diagrams of AFC 9-sampling I/Q-demodulator in comparison with AFC 7-sampling procedure.

$$2a+4b+6b+8a=c+3d+5e+7d+9c\,;$$
$$c-d+e-d+c=0\,. \qquad (24)$$

TABLE II. COEFFICIENTS OF 9-SAMPLING **I/Q-D**EMODULATION

C[1]	C[2]	a	B	c	d	e
3	-1	3	5	1	4	6
4		4	12		8	14
5		5	19		12	22
6		6	26		16	30

The desired coefficients are expressed from (24) through three variables in the following way:

a=C[1]; b=C[1]+2C[2]; c=C[3]; d=C[1]+C[2];
e=2(C[1]+C[2]-C[3]).

The specific value of this system of coefficients is shown in Table III.

TABLE III. COEFFICIENTS OF 9-SAMPLING **I/Q-D**EMODULATION

C[1]	C[2]	C[3]	a	b	c	d	e
1	1	1	1	3	1	2	2
1	2	1	1	5	1	3	4
2	1	1	2	4	1	3	4
3	1	1	3	5	1	4	6
4	1	1	4	6	1	5	8
2	2	1	2	6	1	4	6
4	2	1	4	8	1	6	10
4	4	1	4	12	1	8	14
5	4	1	5	13	1	9	16
5	7	1	5	19	1	12	22
6	10	1	6	26	1	16	30

AFC calculating dependence for the pair of possible 9-sampling demodulators is seen in Fig. 3. For comparison the line "4" presents 7-sampling AFC version in Fig. 2. In Fig.3 number "1" is for diagrams of 7-sampling demodulator AFC with coefficients {a=2; b=4; c=1; d=3}; "2" is for AFC with a set of coefficients {a=5; b=19; c=1; d=12, e=22}; "3" is for AFC of coefficients {a=2; b=6; c=1; d=4; e=6}.

The received results shows that increasing number of independent variables, which are used for coefficient demodulation calculation, allows to decrease range of values spread (dynamic range) of weighting factors.. Nevertheless, it will be observed that the occurrence of AFC minor lobes is typical for the set of coefficients with minimal values of dynamic range, but it is not always acceptable in practice.

Based on examined approaches to coefficient calculation of I/Q-filtration, the author obtained its family for higher order demodulation, part of which is presented in Table IV.

The availability of alternative variants of 11-sampling demodulator in the basis of variables C[1], C[2] in table is determined by various formats, used for its synthesis of exponential equation system. In the mentioned cases the first variant of weighting factors corresponds to monotonically increasing degree of an equation, the second – to the availability of equation with indexes in it, breaking the monotonic dependence.

TABLE IV. GENERAL VIEW OF **I/Q**-FILTRATION COEFFICIENTS WITH ZERO ALTERNATING SUM, EXPRESSED BY INDEPENDENT VARIABLES

I/Q-demodulator order (variables)	General view of integer-valued coefficients
11 (C)	a=8C; b=64C; c=112C; d=C; e=29C; f=98C
11 (C[1], C[2]) 1-й вариант	a=C[1]; b=16C[1]+64C[2]; c=30C[1]+128C[2]; d=-C[2]; e=6C[1]+19C[2]; f=26C[1]+110C[2]
11 (C[1], C[2]) 2-й вариант	a=C[1]; b=-648C[1]+5248C[2]; c=-1298C[1]+10496C[2]; d=10C[1]-79C[2]; e=-190C[1]+1549C[2]; f=-1116C[1]+9026C[2]
11 (C[1], C[2], C[3])	a=C[1], b=4C[2], c=-2C[1]+8C[2], d=C[3], e=2C[1]+C[2]-3C[3]; f=-2C[1]+7C[2]+2C[3]
13 (C)	a=10C; b=130C; c=372C; d=C; e=46C; f=255C; g=420C
13 (C[1], C[2])	a=C[1], b=29C[1]+160C[2], c=98C[1]+608C[2], d=-C[2], e=8C[1]+34C[2]; f=64C[1]+385C[2]; g=112C[1]+700C[2]
13 (C[1], C[2], C[3])	a=C[1]; b=C[1]+2C[2]; c=62C[1]+94C[2]+256C[3]; d=C[1]+C[2]+3C[3]; e= -4C[1]-7C[2]-22C[3]; f=31C[1]+47C[2]+125C[3]; g=72C[1]+110C[2]+300C[3]
15 (C)	a=12C; b=232C; c=1012C; d=1584C; e=C; f=67C; g=561C; h=1419C
15 (C[1], C[2])	a=C[1]; b=46C[1]+320C[2]; c=255C[1]+2048C[2]; d=420C[1]+3456C[2]; e=-C[2]; f=10C[1]+53C[2]; g=130C[1]+999C[2]; h=372C[1]+3045C[2]
15 (C[1], C[2], C[3])	a=C[1]; b=6C[1]+8C[2]; c=447C[1]+64C[2]+512C[3]; d=884C[1]+112C[2]+1024C[3]; e=C[1]+C[3]; f=-8C[1]+C[2]-13C[3]; g=146C[1]+29C[2]+161C[3]; h=757C[1]+98C[2]+875C[3]
15 (C[1], C[2], C[3], C[4])	a=C[1]; b=C[2]; c=31C[1]+16C[2]+64C[3]; d=64C[1]+30C[2]+128C[3]; e=C[4]; f=2C[1]-C[3]-5C[4]; g=6C[1]+6C[2]+19C[3]+9C[4]; h=56C[1]+26C[2]+110C[3]-5C[4]

For example, in the first variant the equations of from 0th to 5th degrees are used:

$$2a+4b+6c+8b+10a=d+3e+5f+7f+9e+11d\ ;$$
$$2^2a+4^2b+6^2c+8^2b+10^2a=d+3^2e+5^2f+7^2f+9^2e+11^2d\ ;$$
$$2^3a+4^3b+6^3c+8^3b+10^3a=d+3^3e+5^3f+7^3f+9^3e+11^3d\ ;$$
$$2^4a+4^4b+6^4c+8^4b+10^4a=d+3^4e+5^4f+7^4f+9^4e+11^4d\ ;$$
$$2^5a+4^5b+6^5c+8^5b+10^5a=d+3^5e+5^5f+7^5f+9^5e+11^5d\ ;$$
$$a-b+c-b+a=0,$$

and in the second – instead of equation of the 4-th and the 7-th degrees we use the equation of the sixth degree in combination with numeration of sampling, starting with zero:

$$2a+4b+6c+8b+10a=d+3e+5f+7f+9e+11d\ ;$$
$$a+3^2b+5^2c+7^2b+9^2a=0d+2^2e+4^2f+6^2f+8^2e+10^2d\ ;$$
$$2^3a+4^3b+6^3c+8^3b+10^3a=d+3^3e+5^3f+7^3f+9^3e+11^3d\ ;$$
$$a+3^6b+5^6c+7^6b+9^6a=0d+2^6e+4^6f+6^6f+8^6e+10^6d\ ;$$
$$a-b+c-b+a=0.$$

If C[1]=6, C[2]=-1 in the first variant of 11-sampling demodulator, we obtain a=6; b=32; c=52; d=1; e=17; f=46, and respectively:

$$I=6u_2-32u_4+52u_6-32u_8+6u_{10};$$
$$Q=u_1-17u_3+46u_5-46u_7+17u_9-u_{11}. \qquad (24)$$

Similarly, if C[1]=5, then C[2]=-1 is a=5; b=16; c=22; d=1; e=11; f=20. The values of variables C[1]<5 if C[2]=-1 are inappropriate in this case, as it causes negative weighting factors.

The second variant of demodulator of 11-th order is characterized by high dynamic range of coefficients. For example, if C[1]=8, C[2]=-1 then weighting factors correspond to the first table row of Table IV (a=8; b=64; c=112; d=1; e=29; f=98).

Analogical effects appear even in case of higher order of the quadrature demodulator.

III. CONCLUSION

The proposed method of analytical calculation of weighting coefficients of I/Q-demodulator of the odd dimension simplifies the problem of its synthesis. Further investigations should be focused on the analysis of phase error of examined demodulators, and also on conducting comparative analysis of its behavior for making recommendations for practical use.

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