THE MATRIX MODELS OF DIGITAL ANTENNA ARRAYS WITH NONIDENTICAL CHANNELS

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The modern technology of radar and mobile communications systems is adaptive digital beam forming. When considering the multicoordinate digital beam forming in radar and communication systems with nonidentical channels of antenna arrays there is a problem of compact matrix record of the receiving channels responses. To solve the given problem it is proposed to operate with a special type of the matrices product, named by the author as "penetrated" and "generalized face-splitting" products.

According to the definition [1], for p×g-matrix A and p×gn-matrix B with p×g-blocks ($B = [B_n]$) their penetrated face-splitting product $A \boxdot B$ is the p×gn-block-matrix $[A \circ B_n]$, where "$\circ$" -a symbol of Adamar splitting, B_n — is a p×g-block of matrix B:

$$A \boxdot B = [A \circ B_1 \mid A \circ B_2 \mid \cdots \mid A \circ B_n \mid \cdots] \text{ or } A \boxdot B = \begin{bmatrix} A \circ B_1 \\ \hline A \circ B_2 \\ \hline \vdots \\ \hline A \circ B_n \\ \hline \vdots \end{bmatrix}.$$

The example:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ \hline B_2 \\ \hline B_3 \end{bmatrix} =$$

$$= \begin{bmatrix} b_{111} & b_{121} \\ b_{211} & b_{221} \\ b_{311} & b_{321} \\ \hline b_{112} & b_{122} \\ b_{212} & b_{222} \\ b_{312} & b_{322} \\ \hline b_{113} & b_{123} \\ b_{213} & b_{223} \\ b_{313} & b_{323} \end{bmatrix}, \quad A \boxdot B = \begin{bmatrix} a_{11} \cdot b_{111} & a_{12} \cdot b_{121} \\ a_{21} \cdot b_{211} & a_{22} \cdot b_{221} \\ a_{31} \cdot b_{311} & a_{32} \cdot b_{321} \\ \hline a_{11} \cdot b_{112} & a_{12} \cdot b_{122} \\ a_{21} \cdot b_{212} & a_{22} \cdot b_{222} \\ a_{31} \cdot b_{312} & a_{32} \cdot b_{322} \\ \hline a_{11} \cdot b_{113} & a_{12} \cdot b_{123} \\ a_{21} \cdot b_{213} & a_{22} \cdot b_{223} \\ a_{31} \cdot b_{313} & a_{32} \cdot b_{323} \end{bmatrix}.$$

As an example, the response of three-coordinate flat digital antenna array of R×R elements can be written down through penetrated face-splitting product of matrices as (without noise):

$$U = \dot{a} \cdot (Q \boxdot F) = \dot{a} \cdot [Q \circ F_1 \mid Q \circ F_2 \mid \cdots \mid Q \circ F_r \mid \cdots],$$

where $\dot{a}$ — is a complex signal amplitude,

$$Q = \begin{bmatrix} \dot{Q}_{11}(x,y) & \dot{Q}_{12}(x,y) & \cdots & \dot{Q}_{1R}(x,y) \\ \dot{Q}_{21}(x,y) & \dot{Q}_{22}(x,y) & \cdots & \dot{Q}_{2R}(x,y) \\ \vdots & \vdots & \vdots & \vdots \\ \dot{Q}_{R1}(x,y) & \dot{Q}_{R2}(x,y) & \cdots & \dot{Q}_{RR}(x,y) \end{bmatrix}$$

is the matrix of the directivity characteristics of primary channels in azimuth and elevation angle planes (can not be factorized),

$$F = \left[\begin{array}{ccc|c|ccc} \dot{F}_{111}(\omega) & \cdots & \dot{F}_{1R1}(\omega) & & \dot{F}_{11G}(\omega) & \cdots & \dot{F}_{1RG}(\omega) \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \dot{F}_{R11}(\omega) & \cdots & \dot{F}_{RR1}(\omega) & & \dot{F}_{R1G}(\omega) & \cdots & \dot{F}_{RRG}(\omega) \end{array}\right]$$

is the block-matrix of amplitude-frequency characteristics meanings $\dot{F}_{nmg}(\omega)$ of G filters for R×R nonidentical receiving channels ($\dot{F}_{11g}(\omega) \neq \dot{F}_{rrg}(\omega)$);

$$Q \circ F_g = \begin{bmatrix} \dot{Q}_{11}(x,y)\dot{F}_{11g}(\omega) & \cdots & \dot{Q}_{1R}(x,y)\dot{F}_{1Rg}(\omega) \\ \dot{Q}_{21}(x,y)\dot{F}_{21g}(\omega) & \cdots & \dot{Q}_{2R}(x,y)F_{2Rg}(\omega) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R1}(x,y)\dot{F}_{R1g}(\omega) & \cdots & \dot{Q}_{RR}(x,y)\dot{F}_{RRg}(\omega) \end{bmatrix},$$

U – block-matrix of voltages of the channels responses.

To select a single source on four coordinates (azimuth, elevation angle, frequency and range) the response of digital antenna array can be written down through generalized face-splitting product or generalized transposed face-splitting product (the theory of face-splitting products is presented in [1–4]). According to the definition, for block-matrices $A = [A_{ij}]$ and $B = [B_{ig}]$ with p×g- blocks their generalized face-splitting product $A \tilde{\square} B$ is the block-matrix

$$[A_{ij} \boxdot [B_{i1} \; B_{i2} \; \cdots \; B_{ig} \; \cdots]] .$$

The example:

$$A \tilde{\square} B = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ A_{P1} & A_{P2} & \cdots & A_{PT} \end{bmatrix} \tilde{\square} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \cdots & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{PG} \end{bmatrix} =$$

$$= \left[\begin{array}{c|c|c} A_{11} \boxdot [B_{11} \; \cdots \; B_{1G}] & \cdots & A_{1T} \boxdot [B_{11} \; \cdots \; B_{1G}] \\ A_{21} \boxdot [B_{21} \; \cdots \; B_{2G}] & \cdots & A_{2T} \boxdot [B_{21} \; \cdots \; B_{2G}] \\ \vdots & \vdots & \vdots \\ A_{P1} \boxdot [B_{P1} \; \cdots \; B_{PG}] & \cdots & A_{PT} \boxdot [B_{P1} \; \cdots \; B_{PG}] \end{array}\right].$$

The alternative to the above considered is a generalized transposed face-splitting block-matrices product:

$$A \tilde{\blacksquare} B = \left[A_{ij} \boxdot \begin{bmatrix} B_{1j} \\ B_{2j} \\ \vdots \\ B_{Gj} \end{bmatrix} \right].$$

For example, it can be written down:

$$A \tilde{\blacksquare} B = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ A_{P1} & A_{P2} & \cdots & A_{PT} \end{bmatrix} \tilde{\blacksquare} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \cdots & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{PG} \end{bmatrix} =$$

$$= \left[\begin{array}{c|c|c|c} A_{11} \boxdot \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{P1} \end{bmatrix} & A_{12} \boxdot \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{P2} \end{bmatrix} & \cdots & A_{1T} \boxdot \begin{bmatrix} B_{1G} \\ B_{2G} \\ \vdots \\ B_{PG} \end{bmatrix} \\ \hline \vdots & \vdots & \cdots & \vdots \\ \hline A_{P1} \boxdot \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{P1} \end{bmatrix} & A_{P2} \boxdot \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{P2} \end{bmatrix} & \cdots & A_{PT} \boxdot \begin{bmatrix} B_{1G} \\ B_{2G} \\ \vdots \\ B_{PG} \end{bmatrix} \end{array} \right].$$

The response of four-coordinate flat digital antenna array with R×R nonidentical channels can be present as (without noise):

$$U = \left(Q \boxdot \left\{ S \tilde{\square} F \right\}\right) \cdot \dot{a} = Q \boxdot \left[S_1 \boxdot F \,\middle|\, S_2 \boxdot F \,\middle|\, \cdots \,\middle|\, S_T \boxdot F \right] \cdot \dot{a},$$

where

$$S = \left[\begin{array}{ccc|c|ccc} S_{111}(z) & \cdots & S_{1R1}(z) & & S_{11T}(z) & \cdots & S_{1RT}(z) \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ S_{R11}(z) & \cdots & S_{RR1}(z) & & S_{R1T}(z) & \cdots & S_{RRT}(z) \end{array} \right]$$

is the matrix of the responses of single signal in T range gates (all channels have nonidentical radio impulse curve $S_{11t}(\omega) \neq S_{rrt}(\omega)$).

The alternate to considered above variant of analytical model of four-coordinate radar with flat digital antenna array is

$$U = \left(Q \boxdot \left\{ \tilde{S} \tilde{\blacksquare} \tilde{F} \right\}\right) \cdot \dot{a} = Q \boxdot \begin{bmatrix} S_1 \boxdot \tilde{F} \\ \hline \vdots \\ \hline S_T \boxdot \tilde{F} \end{bmatrix} \cdot \dot{a},$$

where

$$\tilde{S} = S^R = \left[S_1 \middle| \cdots \middle| S_T \right]^R = \begin{bmatrix} S_1 \\ \hline \vdots \\ \hline S_2 \end{bmatrix} = \left[\begin{array}{ccc} S_{111}(z) & \cdots & S_{1R1}(z) \\ \vdots & \cdots & \vdots \\ S_{R11}(z) & \cdots & S_{RR1}(z) \\ \hline S_{11T}(z) & \cdots & S_{1RT}(z) \\ \vdots & \cdots & \vdots \\ S_{R1T}(z) & \cdots & S_{RRT}(z) \end{array} \right],$$

$$\tilde{F} = F^R = \left[F_1 \middle| \cdots \middle| F_G \right]^R = \begin{bmatrix} F_1 \\ \hline \vdots \\ \hline F_2 \end{bmatrix} = \left[\begin{array}{ccc} F_{111}(\omega) & \cdots & F_{1R1}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R11}(\omega) & \cdots & F_{RR1}(\omega) \\ \hline F_{11G}(\omega) & \cdots & F_{1RG}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R1G}(\omega) & \cdots & F_{RRG}(\omega) \end{array} \right],$$

"R" is the symbol of block-rotation (this new block-matrix operation is proposed by author).

With the considered matrices models, on the basis of Neudecker's matrix derivative [3,4] an information Fischer's block-matrix, describing the accuracy of joint estimation of angular coordinates, range and frequency, is obtained:

$$I = \frac{1}{\sigma^2} \cdot \left[\begin{array}{c|c} P^T \cdot P & a^* \cdot P^T \cdot \dfrac{\partial P}{\partial Y} \\ \hline \dot{a} \cdot \left(\dfrac{\partial P}{\partial Y}\right)^T \cdot P & \left(\dfrac{\partial P}{\partial Y}\right)^T (\dot{a} a^* \cdot 1_{RRTG}) \dfrac{\partial P}{\partial Y} \end{array} \right],$$

where 1_{RRTG} — a unit matrix of dimension R×R×T×G, $\dfrac{\partial P}{\partial Y}$ — Neudecker's derivative of matrix P on vector Y formed of unknown estimations of angular coordinates, range and frequency of sources,

$$P = Q \boxdot \left\{ S \tilde{\square} F \right\} \text{or } P = Q \boxdot \left\{ \tilde{S} \tilde{\blacksquare} \tilde{F} \right\}.$$

To analyse multistatic radar systems the following matrix model (without noise) can be used:

$$U = \left(\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_P \end{bmatrix} \tilde{\square} \begin{bmatrix} S_{11} & \cdots & S_{T1} \\ \vdots & \vdots & \vdots \\ S_{1P} & \cdots & S_{TP} \end{bmatrix} \tilde{\square} \begin{bmatrix} F_{11} & \cdots & F_{G1} \\ \vdots & \vdots & \vdots \\ F_{1P} & \cdots & F_{GP} \end{bmatrix} \right) \cdot \dot{a},$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_P \end{bmatrix} = \left[\begin{array}{ccc} \dot{Q}_{111}(x,y) & \cdots & \dot{Q}_{1R1}(x,y) \\ \dot{Q}_{211}(x,y) & \cdots & \dot{Q}_{2R1}(x,y) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R11}(x,y) & \cdots & \dot{Q}_{RR1}(x,y) \\ \hline \dot{Q}_{11P}(x,y) & \cdots & \dot{Q}_{1RP}(x,y) \\ \dot{Q}_{21P}(x,y) & \cdots & \dot{Q}_{2RP}(x,y) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R1P}(x,y) & \cdots & \dot{Q}_{RRP}(x,y) \end{array} \right],$$

$$S_{tp} = \begin{bmatrix} S_{11tp}(z) & \cdots & S_{1Rtp}(z) \\ \vdots & \cdots & \vdots \\ S_{R1tp}(z) & \cdots & S_{RRtp}(z) \end{bmatrix},$$

$$F_{gp} = \begin{bmatrix} F_{11gp}(\omega) & \cdots & F_{1Rgp}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R1gp}(\omega) & \cdots & F_{RRgp}(\omega) \end{bmatrix},$$

$$U_{gtp} = \left(Q_p \circ S_{tp} \circ F_{gp}\right) \cdot \dot{a},$$

P is a number of radar position.

In the general case, for multiple signals the modeling concept, based on using of block generalized face-splitting product (symbol "$\tilde{\circledcirc}$") and block generalized transposed face-splitting product (symbol "$\tilde{\circledbullet}$") can be proposed. According to the definition,

$$A \,\tilde{\circledcirc}\, B = \left[A_{bg} \,\tilde{\square}\, B_{bk}\right]_{dn}, \quad A \,\tilde{\circledbullet}\, B = \left[A_{bg} \,\tilde{\blacksquare}\, B_{kg}\right]_{dn}.$$

The example:

$$A = \left[\begin{array}{cc|cc} A_{111} & A_{121} & A_{112} & A_{122} \\ A_{211} & A_{221} & A_{212} & A_{222} \end{array}\right],$$

$$B = \left[\begin{array}{cc|cc} B_{111} & B_{121} & B_{112} & B_{122} \\ B_{211} & B_{221} & B_{212} & B_{222} \end{array}\right], \quad A \,\tilde{\circledcirc}\, B =$$

$$= \left[\left[\begin{matrix} A_{111} & A_{121} \\ A_{211} & A_{221} \end{matrix}\right] \tilde{\square} \left[\begin{matrix} B_{111} & B_{121} \\ B_{211} & B_{221} \end{matrix}\right] \,\middle|\, \left[\begin{matrix} A_{112} & A_{122} \\ A_{212} & A_{222} \end{matrix}\right] \tilde{\square} \left[\begin{matrix} B_{112} & B_{122} \\ B_{212} & B_{222} \end{matrix}\right]\right],$$

$$A \,\tilde{\circledbullet}\, B =$$

$$= \left[\left[\begin{matrix} A_{111} & A_{121} \\ A_{211} & A_{221} \end{matrix}\right] \tilde{\blacksquare} \left[\begin{matrix} B_{111} & B_{121} \\ B_{211} & B_{221} \end{matrix}\right] \,\middle|\, \left[\begin{matrix} A_{112} & A_{122} \\ A_{212} & A_{222} \end{matrix}\right] \tilde{\blacksquare} \left[\begin{matrix} B_{112} & B_{122} \\ B_{212} & B_{222} \end{matrix}\right]\right].$$

The model of four-coordinate radar with flat digital antenna array in multisignal case can be written as:

$$U = \left(Q \,\tilde{\circledbullet}\, \left(S \,\tilde{\circledbullet}\, F\right)\right)\left(A \otimes 1_R\right),$$

where A is the vector of complex amplitudes of signals of Ì sources,

$$Q = \left[Q_1 \; Q_2 \cdots Q_M\right], \quad Q_m = \left[\begin{matrix} Q_{11}(x_m, y_m) & \cdots & Q_{1R}(x_m, y_m) \\ \vdots & \cdots & \vdots \\ Q_{R1}(x_m, y_m) & \cdots & Q_{RR}(x_m, y_m) \end{matrix}\right]$$

$$S = \left[S_1 \; S_2 \cdots S_M\right],$$

$$S_m = \left[\begin{matrix} S_{111}(z_m) & \cdots & S_{1R1}(z_m) \\ \vdots & \cdots & \vdots \\ S_{R11}(z_m) & \cdots & S_{RR1}(z_m) \\ \hline \vdots & & \vdots \\ S_{11T}(z_m) & \cdots & S_{1RT}(z_m) \\ \vdots & \cdots & \vdots \\ S_{R1T}(z_m) & \cdots & S_{RRT}(z_m) \end{matrix}\right],$$

$$F = \left[F_1 \; F_2 \cdots F_M\right], \quad F_m = \left[\begin{matrix} F_{111}(\omega_m) & \cdots & F_{1R1}(\omega_m) \\ \vdots & \cdots & \vdots \\ F_{R11}(\omega_m) & \cdots & F_{RR1}(\omega_m) \\ \hline \vdots & & \vdots \\ F_{11G}(\omega_m) & \cdots & F_{1RG}(\omega_m) \\ \vdots & \cdots & \vdots \\ F_{R1G}(\omega_m) & \cdots & F_{RRG}(\omega_m) \end{matrix}\right],$$

1_R is a unit matrix of dimension R; $\otimes$ — symbol of Kronecker's- products of matrixes.

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