ACCURACY OF LINEAR DIGITAL ANTENNA ARRAY AT JOINT
ESTIMATION OF RANGE AND ANGULAR COORDINATE OF M SOURCES

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For the analysis of accuracy of linear digital antenna
array (DAA) it is offered to use the matrix record of
its response of a kind:

\[ U = P \circ A, \]

where \( P = S \circ F, A = [a_1 \ a_2 \ \ldots \ a_M]^T \) – the
vector of complex signals amplitudes,

\[ S = \begin{bmatrix}
  S_1(z_1) & S_1(z_2) & \ldots & S_1(z_M) \\
  S_2(z_1) & S_2(z_2) & \ldots & S_2(z_M) \\
  \vdots & \vdots & \ddots & \vdots \\
  S_T(z_1) & S_T(z_2) & \ldots & S_T(z_M)
\end{bmatrix}, \]

\( T \times M \) - matrix of the responses of \( T \) gates of range
on \( M \) of signals,

\[ F = \begin{bmatrix}
  F_1(x_1) & F_1(x_2) & \ldots & F_1(x_M) \\
  F_2(x_1) & F_2(x_2) & \ldots & F_2(x_M) \\
  \vdots & \vdots & \ddots & \vdots \\
  F_R(x_1) & F_R(x_2) & \ldots & F_R(x_M)
\end{bmatrix}. \]

\( R \times M \)-matrix of meanings of the directivity
characteristics of \( R \) reception channels of DAA in
directions of \( M \) sources; \( \circ \) - symbol of transposed
face-splitting matrixes product (is entered by the
author [1]).

With the account of (1), on the basis of matrix
Neudecker’s derivative [2] an information Fisher’s
block-matrix, describing the accuracy of joint
estimation of range and angular coordinate, is
obtained:

\[ I = \frac{1}{\sigma^2} \times \begin{bmatrix}
  p^T \cdot p & (A^* \otimes p^T) \cdot \frac{\partial P}{\partial Y} \\
  (\frac{\partial P}{\partial Y})^T \cdot (A \otimes p) & (\frac{\partial P}{\partial Y})^T \cdot (A^* \otimes 1_{RT}) \cdot \frac{\partial P}{\partial Y}
\end{bmatrix}, \]

where \( \frac{\partial P}{\partial Y} \) – Neudecker’s derivative of matrix \( P \) on
vector \( Y \), made of unknown estimations of range
and angular coordinates of \( M \) sources; \( 1_{RT} \) – identity
matrix of dimension \( R \times T \); \( \otimes \) – symbol of
Kronecker- products of matrixes.

As in the considered case we are interested only in
dispersions of components of vector \( Y \), it is
necessary to form only the right bottom block of
matrix \( H = I^{-1} \). According to a procedure of the
block-matrix inversion, the necessary block of a
matrix \( H \) will be written down as:

\[ H_{22} = \left[ C \cdot B^* \cdot A^{-1} \cdot B \right]^{-1}, \]

where \( A = p^T \cdot p, B = (A^* \otimes p^T) \cdot \frac{\partial P}{\partial Y}, \)

\[ C = \left( \frac{\partial P}{\partial Y} \right)^T \cdot (A^* \otimes 1_{RT}) \cdot \frac{\partial P}{\partial Y}. \]

It is essential, that for preservation of the
dependence of dispersion of estimations of ranges
and angular coordinates of \( M \) sources on differences
of initial phases of their signals, it is necessary in (2)
to use only the real part of matrix difference:

\[ H_{22} = \left[ \text{Re} \left( C \cdot B^* \cdot A^{-1} \cdot B \right) \right]^{-1}. \]

Unfortunately, the author has not known yet a strict
substantiation of such approach. However, its
neglect results in a situation, when obtained
dispersion of estimations of non-energetic
parameters of signals of \( M \) sources correspond to
only cophase or to anti-phase situations of signals
reception [3].

In case of a single source the problem of
investigation of estimation accuracy is much
simplified. Thus \( A = a_1, Y = [x_1 \ z_1]^T, \)

\[ P = S \circ F = \begin{bmatrix}
  F_1(x_1) \\
  \vdots \\
  F_R(x_1)
\end{bmatrix}. \]
The analysis of resulted relations allows to make a conclusion about the possibility of increase of the accuracy of the solution of range and angle measurement at the expense of minimization of

\[
\left\{ \sum_{r=1}^{R} S_t(z_i) \cdot \frac{\partial R_t(x_i)}{\partial \alpha_1} \cdot \frac{\partial R_t(x_i)}{\alpha_1} \right\} - \left\{ \sum_{r=1}^{R} F_t(x_i) \cdot \frac{\partial F_t(x_i)}{\partial \alpha_1} \right\}
\]

for identical DAA channels or sum

\[
\left\{ \sum_{r=1}^{R} \sum_{t=1}^{T} S_t(z_i) \cdot \frac{\partial R_t(x_i)}{\partial \alpha_1} \cdot \frac{\partial R_t(x_i)}{\partial \alpha_1} \cdot \frac{\partial R_t(x_i)}{\partial \alpha_1} \cdot \frac{\partial R_t(x_i)}{\partial \alpha_1} \right\} - \left\{ \sum_{r=1}^{R} F_t(x_i) \cdot \frac{\partial F_t(x_i)}{\partial \alpha_1} \cdot \frac{\partial F_t(x_i)}{\partial \alpha_1} \right\}
\]

in more general case, when in each t – gate its own set of directivity characteristics of channels is formed and the description of the responses of gates in channels are non-identical.

REFERENCES

1. Slyusar V. I. Analytical model of the digital antenna array on a basis of face-splitting matrix product. - this volume.
