

ACCURACY OF LINEAR DIGITAL ANTENNA ARRAY AT JOINT ESTIMATION OF RANGE AND ANGULAR COORDINATE OF M SOURCES

V. I. Slyusar

Scientific Centre of problems of protection from the precision weapon at Kiev institute of Forces Ground
Kiev, Andruschenko street, 4

For the analysis of accuracy of linear digital antenna array (DAA) it is offered to use the matrix record of its response of a kind:

$$U = P \cdot A, \quad (1)$$

where $P = S \blacksquare F$, $A = [a_1 \ a_2 \ \dots \ a_M]^T$ - the vector of complex signals amplitudes,

$$S = \begin{bmatrix} S_1(z_1) & S_1(z_2) & \dots & S_1(z_M) \\ S_2(z_1) & S_2(z_2) & \dots & S_2(z_M) \\ \vdots & \vdots & \vdots & \vdots \\ S_T(z_1) & S_T(z_2) & \dots & S_T(z_M) \end{bmatrix}$$

$T \times M$ - matrix of the responses of T gates of range on M of signals,

$$F = \begin{bmatrix} F_1(x_1) & F_1(x_2) & \dots & F_1(x_M) \\ F_2(x_1) & F_2(x_2) & \dots & F_2(x_M) \\ \vdots & \vdots & \vdots & \vdots \\ F_R(x_1) & F_R(x_2) & \dots & F_R(x_M) \end{bmatrix}$$

$R \times M$ -matrix of meanings of the directivity characteristics of R reception channels of DAA in directions of M sources; \blacksquare - symbol of transposed face-splitting matrixes product (is entered by the author [1]).

With the account of (1), on the basis of matrix Neudecker's derivative [2] an information Fisher's block-matrix, describing the accuracy of joint estimation of range and angular coordinate, is obtained:

$$I = \frac{1}{\sigma^2} \times \begin{bmatrix} P^T \cdot P & \vdots & (A^* \otimes P^T) \cdot \frac{\partial P}{\partial Y} \\ \dots & \dots & \dots \\ \left(\frac{\partial P}{\partial Y}\right)^T \cdot (A \otimes P) & \vdots & \left(\frac{\partial P}{\partial Y}\right)^T \cdot (AA^* \otimes I_{RT}) \cdot \frac{\partial P}{\partial Y} \end{bmatrix},$$

where $\frac{\partial P}{\partial Y}$ - Neudecker's derivative of matrix P on vector Y , made of unknown estimations of range and angular coordinates of M sources; I_{RT} - identity matrix of dimension $R \times T$; \otimes - symbol of Kronecker-products of matrixes.

As in the considered case we are interested only in dispersions of components of vector Y , it is

necessary to form only the right bottom block of matrix $H = I^{-1}$. According to a procedure of the block-matrix inversion, the necessary block of a matrix H will be written down as:

$$H_{22} = [C - B^* \cdot A^{-1} \cdot B]^{-1}, \quad (2)$$

where $A = P^T \cdot P$, $B = (A^* \otimes P^T) \cdot \frac{\partial P}{\partial Y}$,

$$C = \left(\frac{\partial P}{\partial Y}\right)^T \cdot (AA^* \otimes I_{RT}) \cdot \frac{\partial P}{\partial Y}.$$

It is essential, that for preservation of the dependence of dispersion of estimations of ranges and angular coordinates of M sources on differences of initial phases of their signals, it is necessary in (2) to use only the real part of matrix difference:

$$H_{22} = \left[\text{Re} \left\{ C - B^* \cdot A^{-1} \cdot B \right\} \right]^{-1}.$$

Unfortunately, the author has not known yet a strict substantiation of such approach. However, its neglectation results in a situation, when obtained dispersion of estimations of non-energetic parameters of signals of M sources correspond to only cophase or to anti-phase situations of signals reception [3].

In case of a single source the problem of investigation of estimation accuracy is much simplified. Thus $A = a_1$, $Y = [x_1 \ z_1]^T$,

$$P = S \blacksquare F = \begin{bmatrix} S_1(z_1) \cdot \begin{bmatrix} F_1(x_1) \\ \vdots \\ F_R(x_1) \end{bmatrix} \\ \vdots \\ S_T(z_1) \cdot \begin{bmatrix} F_1(x_1) \\ \vdots \\ F_R(x_1) \end{bmatrix} \end{bmatrix};$$

$$\frac{\partial \mathcal{P}}{\partial \mathcal{Y}} = \begin{bmatrix} S_1(z_1) \cdot \begin{bmatrix} \frac{\partial F_1(x_1)}{\partial \alpha_1} \\ \vdots \\ \frac{\partial F_R(x_1)}{\partial \alpha_1} \end{bmatrix} & \frac{\partial \mathcal{S}_1(z_1)}{\partial z_1} \cdot \begin{bmatrix} F_1(x_1) \\ \vdots \\ F_R(x_1) \end{bmatrix} \\ \vdots & \vdots \\ S_T(z_1) \cdot \begin{bmatrix} \frac{\partial F_1(x_1)}{\partial \alpha_1} \\ \vdots \\ \frac{\partial F_R(x_1)}{\partial \alpha_1} \end{bmatrix} & \frac{\partial \mathcal{S}_T(z_1)}{\partial z_1} \cdot \begin{bmatrix} F_1(x_1) \\ \vdots \\ F_R(x_1) \end{bmatrix} \end{bmatrix};$$

$$P^T \cdot P = \sum_{t=1}^T S_t^2(z_1) \cdot \left\{ \sum_{r=1}^R F_r^2(x_1) \right\};$$

$$(A^* \otimes P^T) \cdot \frac{\partial \mathcal{P}}{\partial \mathcal{Y}} =$$

$$= a^* \cdot \left[\sum_{t=1}^T S_t^2(z_1) \cdot \left\{ \sum_{r=1}^R F_r^2(x_1) \cdot \frac{\partial F_r(x_1)}{\partial \alpha_1} \right\} \right];$$

$$\left[\sum_{r=1}^R S_t^2(z_1) \cdot \frac{\partial \mathcal{S}_t(z_1)}{\partial z_1} \right] \left\{ \sum_{r=1}^R F_r^2(x_1) \right\};$$

$$\left(\frac{\partial \mathcal{P}}{\partial \mathcal{Y}} \right)^T \cdot (A \otimes P) =$$

$$= a \cdot \left[\sum_{t=1}^T S_t^2(z_1) \cdot \left\{ \sum_{r=1}^R F_r^2(x_1) \cdot \frac{\partial F_r(x_1)}{\partial \alpha_1} \right\} \right];$$

$$\left(\frac{\partial \mathcal{P}}{\partial \mathcal{Y}} \right)^T \cdot (AA^* \otimes 1_{RT}) \cdot \frac{\partial \mathcal{P}}{\partial \mathcal{Y}} =$$

$$a^2 \cdot \left[\sum_{t=1}^T S_t^2(z_1) \cdot \left\{ \sum_{r=1}^R \left(\frac{\partial F_r(x_1)}{\partial \alpha_1} \right)^2 \right\} \right];$$

$$\left[\sum_{r=1}^T S_t(z_1) \cdot \frac{\partial \mathcal{S}_t(z_1)}{\partial z_1} \right] \left\{ \sum_{r=1}^R F_r(x_1) \cdot \left(\frac{\partial F_r(x_1)}{\partial \alpha_1} \right) \right\};$$

$$\left[\sum_{r=1}^T \left(\frac{\partial \mathcal{S}_t(z_1)}{\partial z_1} \right)^2 \right] \left\{ \sum_{r=1}^R F_r^2(x_1) \right\};$$

The analysis of resulted relations allows to make a conclusion about the possibility of increase of the accuracy of the solution of range and angle measurement at the expense of minimization of

$$\text{value} \left\{ \sum_{r=1}^T S_t(z_1) \cdot \frac{\partial \mathcal{S}_t(z_1)}{\partial z_1} \right\} \left\{ \sum_{r=1}^R F_r(x_1) \cdot \left(\frac{\partial F_r(x_1)}{\partial \alpha_1} \right) \right\} -$$

for identical DAA channels or sum

$$\left\{ \sum_{r=1}^R \sum_{t=1}^T S_t(z_1) \cdot \frac{\partial \mathcal{S}_t(z_1)}{\partial z_1} \cdot F_r(x_1) \cdot \left(\frac{\partial F_r(x_1)}{\partial \alpha_1} \right) \right\} - \text{in more}$$

general case, when in each t - gate its own set of directivity characteristics of channels is formed and the description of the responses of gates in channels are non-identical.

REFERENCES

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